Heathfield Community College A Level Maths and Further Maths Summer work

Instructions

- Work on A4 lined paper
- Write out the question and show all your working.
- You need to do a minimum of 3 questions from each exercise to demonstrate understanding.
- You must show all working
- You must mark your own work from the answers provided.
- When you have finished you must fill in the check list.

Advice

You might find a GCSE higher tier textbook helpful. If any section causes problems the "www.mymaths.co.uk" website is very useful. The booster sections "GCSE booster 6 and 7" and "GCSE booster 8 and 9" especially good.

Calculators

You will **need** get the Casio fx991 EX Classwiz plus calculator (or equivalent), as it has statistical tables in it you will need in A level Maths – these are not in the Formulae Booklet.

CHECK LIST

TOPIC		I AM FINE ON THIS TOPIC	I NEED TO DO SOME MORE PRACTICE	I *MUST* GET HELP AT THE BEGINNING OF TERM
Surds				
Factorising	**Difference of 2 Squares**			
	$x^2 + bx + c$			
	$ax^2 + bx + c$			
Completing the square				
Solving Quadratic Equations	**Factorising**			
	Formula			
Linear Simultaneous Equations				
Non-Linear Simultaneous Equations				

SURDS

Rationalising the Denominator

Examples

1. $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ i.e., multiply top and bottom by √3

$= \frac{\sqrt{3}}{3}$

2. $\frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{4\sqrt{2}\times\sqrt{2}}$

$= \frac{\sqrt{2}}{4\times 2}$

$= \frac{\sqrt{2}}{8}$

Exercise 1

Rationalise the denominators:

1. $\frac{1}{\sqrt{2}}$

2. $\frac{1}{\sqrt{7}}$

3. $\frac{7}{\sqrt{5}}$

4. $\frac{\sqrt{2}}{3\sqrt{3}}$

5. $\frac{\sqrt{8}}{\sqrt{32}}$

6. $\frac{\sqrt{5}}{\sqrt{45}}$

7. $\frac{\sqrt{3}}{\sqrt{21}}$

8. $\frac{\sqrt{11}}{\sqrt{132}}$

FACTORISING

Difference of Two Squares

Example 1

Factorise $x^2 - 9$

$x^2 - 9 = (x - 3)(x + 3)$

Example 2

Factorise $9x^2 - 16$

$9x^2 - 16 = (3x)^2 - (4)^2$
$= (3x - 4)(3x + 4)$

Example 3

Factorise $8x^2 - 2$

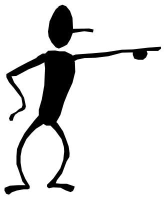
$8x^2$ and 2 are not perfect squares, but 2 is a common factor of the expression.

$8x^2 - 2 = 2(4x^2 - 1)$

$4x^2 - 1 = (2x)^2 - 1^2$ which is the difference of two squares

$\therefore\ 8x^2 - 2 = 2(2x - 1)(2x + 1)$

(*Note*: If a quadratic has a common factor, always take out the common factor before factorising the quadratic into two brackets.)



Exercise 2

Factorise:

1. $x^2 - 1$
2. $x^2 - 16$
3. $x^2 - 25$
4. $4x^2 - 9$
5. $9x^2 - 1$
6. $16x^2 - 25$
7. $49 - x^2$
8. $36 - 25x^2$
9. $25 - 49x^2$
10. $2x^2 - 8$
11. $9x^2 - 36$
12. $12x^2 - 75$

Factorising Quadratic Expressions

When the coefficient of x^2 is unity

Example 1

Factorise $x^2 + 5x + 6$

Reversing the process of multiplying out brackets, we can see that

$x^2 + 5x + 6 = (x + ?)\ (x + ?)$

and 6 must factorise as 6×1 or 3×2.

The key to factorising the quadratic expression is the x term.

$5x$ is the sum of two x terms and the coefficients of these two terms must be the factors of 6.

$3 \times 2 = 6$ and $3x + 2x = 5x$

$\therefore\ x^2 + 5x + 6 = (x + 3)\ (x + 2)$

Example 2

Factorise $x^2 - x - 6$

The factors of -6, when added together, must equal the coefficient of x, which is -1.

The factors of -6 are -6×1, 6×-1, -3×2, 3×-2.
Only $-3 + 2 = -1$

$\therefore\ x^2 - x - 6 = (x - 3)(x + 2)$

Example 3

Factorise $x^2 + 6x$

This is a much simpler quadratic expression to factorise than the standard type because it has a common factor which is x.

$x^2 + 6x = x(x + 6)$

Exercise 3

Factorise the following:

1. $x^2 + 8x + 7$
2. $x^2 + 7x + 10$
3. $x^2 + x - 6$
4. $x^2 - 5x + 6$
5. $x^2 - 7x$
6. $x^2 + 2x - 8$
7. $x^2 - 2x - 15$
8. $x^2 - 6x + 9$
9. $2x^2 + 6x$
10. $x^2 - 4x - 12$
11. $x^2 - 7x + 12$
12. $x^2 + 11x - 12$
13. $6x^2 - 3x$
14. $x^2 - 5x - 6$
15. $x^2 - 6x - 16$
16. $x - 2x^2$
17. $x^2 + 12x + 36$
18. $x^2 - 8x + 16$

When the coefficient of x^2 is not unity

<u>Example</u>

Factorise $2x^2 + 11x + 12$

The method previously used must be adapted to take into account the coefficient 2.

As 2 will only factorise as 2 × 1, it can be see that

$2x^2 + 11x + 12 = (2x + a)(x + b)$

As before, $a \times b = 12$, but now we require

$a + 2b = 11$ (not $a + b$ as before)

Factors of 12 are $a \times b$	$= 1 \times 12$	and $a + 2b$	$= 1 + 24 = 25$
	2×6		$= 2 + 12 = 14$
	3×4		$= 3 + 8 = 11$
	4×3		$= 4 + 6 = 10$
	etc.		

$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$

Alternatively list all possible solutions and select the one which 'works'.

$(2x + 1)(x + 12) = 2x^2 + 25x + 12$
$(2x + 2)(x + 6) = 2x^2 + 14x + 12$
$(2x + 3)(x + 4) = 2x^2 + 11x + 12$

<u>Exercise 4</u>

Factorise:

1. $2x^2 + 5x + 2$
2. $2x^2 + 7x + 6$
3. $2x^2 + 9x - 5$
4. $2x^2 - 13x - 7$
5. $3x^2 + 8x + 5$
6. $3x^2 - 6x - 9$
7. $3x^2 - 8x + 4$
8. $3x^2 - 13x - 10$
9. $2x^2 - 11x + 12$
10. $3x^2 + 20x + 12$
11. $3x^2 - 22x - 16$
12. $2x^2 - 3x - 14$

COMPLETING THE SQUARE

Some expressions can be factorised as $(x+a)^2$ or $(x-a)^2$

e.g. $x^2 + 6x + 9 = (x+3)^2$ - check that you agree!!

$x^2 - 10x + 25 = (x-5)^2$

These expressions are called perfect squares.

For expressions which are not perfect squares, we 'complete the square', which means adjusting the constant term:

<u>Example 1</u>

Express $x^2 + 6x + 11$ in the completed square form $(x+a)^2 + b$.

Firstly, halve the coefficient of the *x* term for inside the bracket:

so we have $(x+3)^2$

If we multiply out, this gives $x^2 + 6x + 9$ so we need to add 2 to obtain $x^2 + 6x + 11$.

So $x^2 + 6x + 11 = (x+3)^2 + 2$

<u>Example 2</u>

Express $x^2 - 10x + 13$ in the completed square form $(x-a)^2 + b$.

Again, halve the coefficient of the *x* term to give $(x-5)^2$

If we multiply out, this gives $x^2 - 10x + 25$ so we need to subtract 12 to obtain $x^2 - 10x + 13$.

So $x^2 - 10x + 13 = (x-5)^2 - 12$

In general terms, the formula for completing the square for $x^2 + px + q$ is:

$$\left(x + \frac{1}{2}p\right)^2 - \left(\frac{1}{2}p\right)^2 + q$$

<u>Exercise 5</u>

Write the following in completed square form:

1. $x^2 + 8x + 7$
2. $x^2 + 6x + 10$
3. $x^2 + 2x - 6$
4. $x^2 - 4x + 6$
5. $x^2 - 8x$
6. $x^2 - 2x - 15$
7. $x^2 - 10x + 9$
8. $x^2 + 6x - 5$
9. $x^2 - 4x - 12$
10. $x^2 - 8x + 11$
11. $x^2 - 3x$
12. $x^2 + 12x + 100$
13. $x^2 - 6x - 16$
14. $x^2 + 5x + 7$
15. $x^2 + 11x + 30$

SOLVING QUADRATIC EQUATIONS

1. By Factorising

Example 1

Solve the equation $x^2 - 5x - 14 = 0$

$$x^2 - 5x - 14 = 0$$

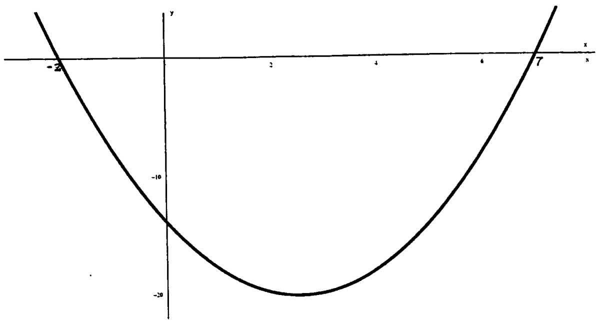
Factorising gives: $(x - 7)(x + 2) = 0$

$\therefore$ either $(x - 7) = 0$ or $(x + 2) = 0$

$\therefore$ either $x = 7$ or $x = -2$

$\therefore$ The solution is $x = 7$ or -2

This can be shown graphically:



Note: A quadratic equation always has *two* possible solutions. If the quadratic is a perfect square, the two solutions will be the same and they are called repeated roots.

Example 2

Solve the equation $2x^2 - 5x + 3 = 0$

$$2x^2 - 5x + 3 = 0$$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $(2x - 3) = 0$ or $x - 1 = 0$

$2x = 3$

$\therefore x = 1^1/_2$ or $x = 1$

Exercise 6

Solve the following equations:

1. $x^2 - 5x + 4 = 0$
2. $x^2 + 11x + 28 = 0$
3. $x^2 - 49 = 0$
4. $x^2 + 6x + 9 = 0$
5. $2x^2 + 7x + 3 = 0$
6. $x^2 + 3x = 0$
7. $3x^2 - 2x - 8 = 0$
8. $5x^2 - 2x = 0$
9. $3x^2 - 20x + 12 = 0$
10. $2x^2 + 3x - 14 = 0$
11. $2x^2 - 2x - 40 = 0$
12. $6x^2 + 3x - 3 = 0$

2. Using the Formula

To solve $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Solve $x^2 + 3x + 1 = 0$

Compare $\quad x^2 + 3x + 1 = 0$
with $\quad ax^2 + bx + c = 0$

Then $a = +1$, $b = +3$, $c = +1$
and substituting for *a, b* and *c* in the formula gives

$$x = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times 1}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

AS you can't use a calculator in C1, answers must be left in surd form

Example 2

Solve $2x^2 - 3x - 1 = 0$

In this equation, $a = 2$, $b = -3$, $c = -1$ and substituting in the formula gives:

$$x = \frac{+3 \pm \sqrt{9 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$x = \frac{3 \pm \sqrt{17}}{4}$$

Exercise 7

Solve the following equations using the quadratic formula.

1. $2x^2 + 2x - 3 = 0$
2. $2x^2 + 4x + 1 = 0$
3. $x^2 + 2x - 2 = 0$
4. $3x^2 - x - 1 = 0$
5. $x^2 + 6x - 10 = 0$
6. $x^2 - 7x + 9 = 0$
7. $4x^2 - 8x - 16 = 0$
8. $3x^2 - 6x + 2 = 0$
9. $2x^2 + 5x = 6$
10. $3x^2 - 10x = -5$
11. $x(x + 2) = 5$
12. $x(x - 3) = -1$

Solving Simultaneous Equations

1. Linear

Example

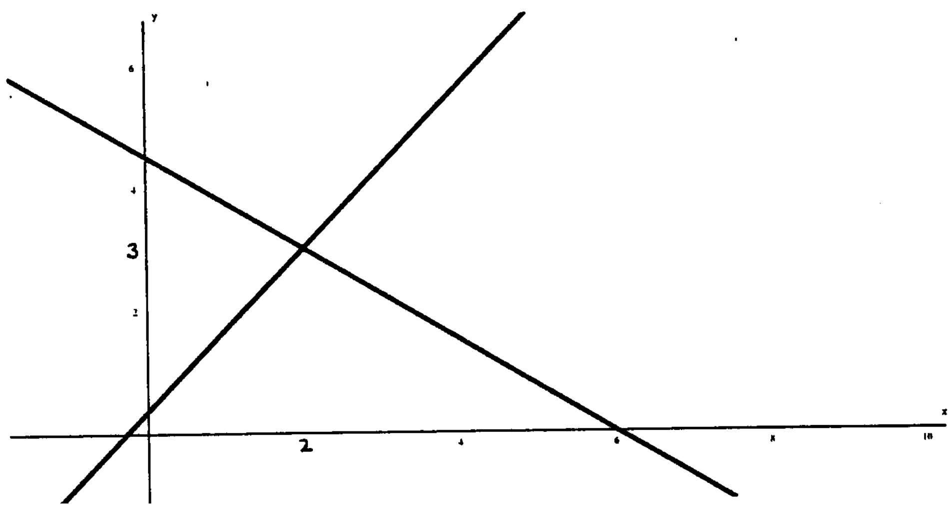
Solve the equations $3x + 4y = 18$ (1)

$4x - 3y = -1$ (2)

Neither *x* nor *y* has the same coefficient in each equation, so this needs to be remedied first.

Method	Working
i) Decide which variable is to be eliminated.	Eliminate *y*
ii) Multiply one or both equations so that this variable has the same coefficient in each equation (not counting the signs).	Multiply equation (1) by 3 and equation (2) by 4 $9x + 12y = 54$ $16x - 12y = -4$
iii) Add or subtract the equations, depending on the signs of the variable to be eliminated.	As the coefficients of *y* have opposite signs, add the equations: $9x + 12y = 54$ $16x - 12y = -4$ $25x = 50$
iv)Solve for the remaining variable.	$x = 2$
v) Substitute into one of the original equations to find the eliminated variable and hence the complete solution.	Substitute $x = 2$ into (1) $3 \times 2 + 4y = 18$ $4y = 12$ $y = 3$ Solution is $x = 2$, $y = 3$
vi) Substitute into the other original equation to check the solution.	Substitute into (2) LHS = $4 \times 2 - 3 \times 3$ $= 8 - 9$ $= -1$ = RHS

You can see this on the following graph:



Exercise 8

Solve the following pairs of simultaneous equations:

1. $2x + y = 8$
   $x + y = 6$

2. $x + 3y = 12$
   $x = y = 10$

3. $x + y = 12$
   $x - y = 2$

4. $3x - 2y = 9$
   $x + 2y = 15$

5. $x + 3y = 6$
   $2x + y = 7$

6. $4x + 2y = 10$
   $3x + 5y = 11$

7. $3x + 2y = 18$
   $4x - y = 2$

8. $2x - 3y = 10$
   $3x + 4y = 15$

9. $x - y = -1$
   $2x + 3y = 28$

10. $7x + y = 9$
    $3x + y = -3$

11. $2x + 5y = 15\frac{1}{2}$
    $3x - 4y = -5\frac{1}{2}$

12. $6x + 5y = -17$
    $3x + 2y = -8$

2. One Linear One Non-Linear

Using Substitution method

<u>Example 1</u>

$y = 2x^2 + 5x - 3$

$5x - y + 5 = 0$

Stage 1: Make x or y the subject of the linear equation

i.e., $5x - y + 5 = 0$

$\therefore$ $y = 5x + 5$

It's easier to make y the subject here

Stage 2: Substitute this rearranged equation into the other one

$y = 2x^2 + 5x - 3$

$\therefore$ $5x + 5 = 2x^2 + 5x - 3$

Stage 3: Rearrange and solve the quadratic

$0 = 2x^2 - 8$

$0 = x^2 - 4$

$0 = (x - 2)(x + 2)$

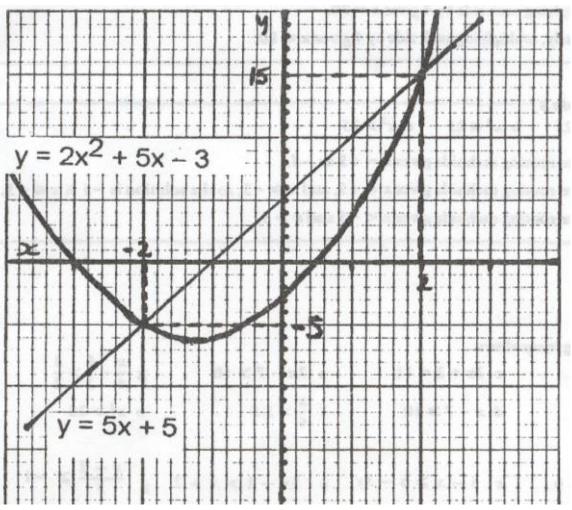
$\therefore$ $x = 2$ or $x = -2$

Stage 4: Find the corresponding values of y using the simplest equation

$x = 2$ $y = 5 \times 2 + 5 = 15$

$x = -2$ $y = 5 \times -2 + 5 = -5$

You can see this on the following graph.



Example 2

Solve simultaneously $y - x = 2$ (1)
$x^2 + y^2 = 10$ (2)

$y - x = 2$ is the linear equation so re=write as $y = x + 2$

Now substitute for y in the non-linear equation

$x^2 + (x + 2)^2 = 10$ which is a quadratic in x only

$x^2 + x^2 + 4x + 4 = 10$

$2x^2 + 4x - 6 = 0$

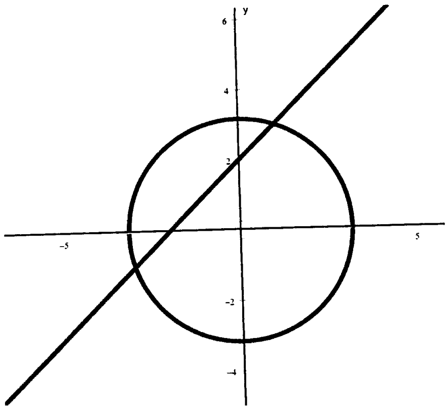
$x + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$ so $x = -3$ or 1

Substitute $x = -3$ in (1) $y + 3 = 2$ therefore $y = -1$

Substitute $x = 1$ in (1) $y - 1 = 2$ therefore $y = 3$

So the solutions are $\underline{x = -3,\ y = -1}$ or $\underline{x = 1,\ y = 3}$



CHECK: in equation (2):

$(-3)^2 + (-1)^2 = 9 + 1 = 10$

$1^2 + 3^2 = 1 + 9 = 10$

Exercise 9

Solve the simultaneous equations:

1. $x + y = 1$
$16x^2 + y^2 = 65$

2. $2x + y = 1$
$x^2 + y^2 = 1$

3. $y - x = 2$
$2x^2 + 3xy + y^2 = 8$

4. $x - 2y = 7$
$x^2 + 4y^2 = 37$

5. $x + y = 9$
$x^2 - 3xy + 2y^2 = 0$

6. $x = 2y$
$x^2 + 3xy = 10$

7. $y = x^2 + 3$
$y = 4x$

8. $u - v = 3$
$u^2 + v^2 = 89$

9. $x + 2y = -3$
$x^2 - 2x + 3y^2 = 11$

10. $y - x = 4$
$2x^2 + xy + y^2 = 8$

ANSWERS

Exercise 1

1. $\frac{\sqrt{2}}{2}$ 2. $\frac{\sqrt{7}}{7}$ 3. $\frac{7\sqrt{5}}{5}$ 4. $\frac{\sqrt{6}}{9}$ 5. ½ 6. 1/3

7. $\frac{\sqrt{7}}{7}$ 8 $\frac{\sqrt{3}}{6}$

Exercise 2

1.	$(x-1)(x+1)$	**5.**	$(3x-1)(3x+1)$	**9.**	$(5-7x)(5+7x)$
2.	$(x-4)(x+4)$	**6.**	$(4x-5)(4x+5)$	**10.**	$2(x-2)(x+2)$
3.	$(x-5)(x+5)$	**7.**	$(7-x)(7+x)$	**11.**	$9(x-2)(x+2)$
4.	$(2x-3)(2x+3)$	**8.**	$(6-5x)(6+5x)$	**12.**	$3(2x-5)(2x+5)$

Exercise 3

1.	$(x+1)(x+7)$	**7.**	$(x+3)(x-5)$	**13.**	$3x(2x-1)$
2.	$(x+2)(x+5)$	**8.**	$(x-3)(x-3)$	**14.**	$(x-6)(x+1)$
3.	$(x-2)(x+3)$	**9.**	$2x(x+3)$	**15.**	$(x-8)(x+2)$
4.	$(x-2)(x-3)$	**10.**	$(x-6)(x+2)$	**16.**	$x(1-2x)$
5.	$x(x-7)$	**11.**	$(x-3)(x-4)$	**17.**	$(x+6)(x+6)$
6.	$(x+4)(x-2)$	**12.**	$(x+12)(x-1)$	**18.**	$(x-4)(x-4)$

Exercise 4

1.	$(2x+1)(x+2)$	**5.**	$(3x+5)(x+1)$	**9.**	$(2x-3)(x-4)$
2.	$(2x+3)(x+2)$	**6.**	$3(x+1)(x-3)$	**10.**	$(3x+2)(x+6)$
3.	$(2x-1)(x+5)$	**7.**	$(3x-2)(x-2)$	**11.**	$(3x+2)(x-8)$
4.	$(2x+1)(x-7)$	**8.**	$(3x+2)(x-5)$	**12.**	$(2x-7)(x+2)$

Exercise 5

1.	$(x+4)^2-9$	**6.**	$(x-1)^2-16$	**11.**	$\left(x-\frac{3}{2}\right)^2-\frac{9}{4}$
2.	$(x+3)^2+1$	**7.**	$(x-5)^2-16$	**12.**	$(x+6)^2+64$
3.	$(x+1)^2-7$	**8.**	$(x+3)^2-14$	**13.**	$(x-3)^2-25$
4.	$(x-2)^2+2$	**9.**	$(x-2)^2-16$	**14.**	$\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$
5.	$(x-4)^2-16$	**10.**	$(x-4)^2-5$	**15.**	$\left(x+\frac{11}{2}\right)^2-\frac{1}{4}$

Exercise 6

1. $x = 4$ or 1	**5.** $x = -3$ or $-{}^{1}/_{2}$	**9.** $x = {}^{2}/_{3}$ or 6
2. $x = -7$ or -4	**6.** $= 0$ or -3	**10.** $x = -{}^{7}/_{2}$ or 2
3. $x = 7$ or -7	**7.** $x = -{}^{4}/_{3}$ or 2	**11.** $x = 5$ or -4
4. $x = -3$ or -3	**8.** $x = 0$ or ${}^{2}/_{5}$	**12.** $x = {}^{1}/_{2}$ or -1

Exercise 7

1. $\dfrac{-2\pm\sqrt{28}}{4} = \dfrac{-1\pm\sqrt{7}}{2}$ note: can cancel by 2

2. $\dfrac{-2\pm\sqrt{2}}{2}$

3. $-1\pm\sqrt{3}$

4. $\dfrac{1\pm\sqrt{13}}{6}$

5. $-3\pm\sqrt{19}$

6. $\dfrac{7\pm\sqrt{13}}{2}$

7. $1\pm\sqrt{5}$

8. $\dfrac{3\pm\sqrt{3}}{3}$

9. $\dfrac{-5\pm\sqrt{73}}{4}$

10. $\dfrac{5\pm\sqrt{10}}{3}$

11. $-1\pm\sqrt{6}$

12. $\dfrac{3\pm\sqrt{5}}{2}$

Exercise 8

1. $x = 2$ $y = 4$	**5.** $x = 3$ $y = 1$	**9.** $x = 5$ $y = 6$
2. $x = 9$ $y = 1$	**6.** $x = 2$ $y = 1$	**10.** $x = 3$ $y = -12$
3. $x = 7$ $y = 5$	**7.** $x = 2$ $y = 6$	**11.** $x = 1{}^{1}/_{2}$ $y = 2{}^{1}/_{2}$
4. $x = 6$ $y = 4{}^{1}/_{2}$	**8.** $x = 5$ $y = 0$	**12.** $x = -2$ $y = -1$

Exercise 9

1. $(2, -1)$, $(-{}^{32}/_{17}, {}^{49}/_{17})$	**5.** $(6, 3)$, $(4{}^{1}/_{2}, 4{}^{1}/_{2})$	**9.** $(1, -2)$, $(-2{}^{3}/_{7}, -{}^{2}/_{7})$
2. $(0, 1)$, $({}^{4}/_{5}, -{}^{3}/_{5})$	**6.** $(2, 1)$, $(-2, -1)$	**10.** $(-1, 3)$, $(-2, 2)$
3. $(-2, 0)$, $({}^{1}/_{3}, {}^{7}/_{3})$	**7.** $(3, 12)$, $(1, 4)$	
4. $(1, -3)$, $(6, -{}^{1}/_{2})$	**8.** $(8, +5)$,$(-5, -8)$	